

Parametric Effects on a Heat Sink with Branched Fins under Natural Convection

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Abstract: This paper presents to determine of the optimum values of the design parameters in a cylindrical heat sink with branched fins. Investigations on the effect of the design parameters, such as the number of fins, length of fin, height of fin, and the outer diameter of the heat sink on heat transfer are reported here. In this analysis, branch angle ($\alpha = 300$) is considered. The Taguchi method, a powerful tool to design optimization, is applied for the tests and standard L9 orthogonal array (OA) with three factors and three levels for each factor are selected. Nine test samples are analyzed in which the total heat transfer rate for each test sample is found. Contribution ratios for each parameter are also found. The results obtained from this analysis are employed to find the optimum design parameter values which are relating to the heat sink performance. The reliability of the optimum test samples is verified. Also, the variation of the average heat transfer rate of optimum sample is reported, when it compared with the reference sample.

Keywords: Cylindrical Heat Sink, natural convection, heat transfer, Taguchi method, optimization.

1 INTRODUCTION

Owing to the higher energy efficiency and the longer lifespan of the light emitting diode (LED) lights, this system has been emerging rapidly in illumination industry which replaces many other traditional light sources. Although the application of this system creates a thermal problem, if the power supply to the LEDs is high which results the heat generated by this system is more, and it reduces the energy efficiency and the service life. To overcome this problem, an efficient heat sink design is required. Therefore, a cooling technology is required for the heat dissipation from the LEDs system to the ambient by using a cylindrical heat sink. The optimization of the cylindrical heat sink is essential to find cooling performance in optimum manner in the LED lighting applications.

There have been various studies on cooling relating to natural convection heat transfer in a cylindrical heat sink

[1,2]. In LED lighting applications, several recent research paper have presented on cylindrical heat sink. Yu et al. [3,4,5] and Jang et al. [6,7] investigated the thermo-flow characteristics like a chimney flow pattern around a radial heat sink under natural convection, and optimization of cooling performance parameter and mass was presented. Jeng et al. [8] developed a Nusselt number correlation of combined convection around the cylindrical heat sink with motor fan, and he reported the effect of fans at various assemblies. Jang et al. [9,10] developed a Nusselt number correlation and observed the orientation effect of a cylindrical heat sink. Also, they observed cross-cut cylindrical heat sink to improve the orientation effects of conventional plate-fin cylindrical heat sink for applications in LED light bulbs. The rate of heat transfer from square vertical fins connected to a horizontal tube found from an experimental based naphthalene sublimation method and reported the effect of fin-to-fin spacing on heat transfer rate by Sparrow and Bahrami [11]. Chen and chou developed a Nusselt number correlation, and they investigated numerically the heat transfer coefficient of square vertical fins attached to a horizontal tube. Also, they reported the effect of fin-to-fin spacing on the heat transfer coefficient [12]. Yildiz and Yüncü performed an experiment to observe the heat transfer rate from annular fin arrays

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attached to a horizontal cylinder, and they optimized the fin-to-fin spacing [13]. Hahne and Zhu proposed the correlation of a Nusselt number for an annular fin attached with horizontal cylinders regarding an experimental study [14]. An experimental study for the Nusselt number correlation for finding natural convection heat transfer from vertical cylinders with plate fins was investigated [15]. Bejan and Almogbel reported to maximize the global thermal conductance, and optimization of a T-shaped fin assembly was performed [16]. Lorenzini and Rocha observed to optimize a Y-shaped fin assembly for minimizing the global thermal resistance, and also for a T-Y fin assembly, they reported to maximize heat removal regarding an optimization of this assembly [17,18]. Kobus and Oshio observed a theoretical and experimental study on a heat sink, and they reported the effect of various geometries on the thermal resistance of a heat sink [19]. Huang et al. [20] found a high heat transfer coefficient at upward and sideward orientation of a square pin fin under natural convection heat transfer. Zografos and Sunderland observed that the heat transfer performance of inline pin fin arrays arrangement showed better than the staggered pin fin arrays [21]. Sparrow and Vemuri reported that the heat transfer coefficient occurred the highest at upward facing orientation of fin and the optimization of the fin density and its height was also investigated [22]. Inada et al. [23] conducted an experiment to observe the effect of flow orientation, the effect of vertical fins with constant height on the heat transfer rate, and also they found an increased heat transfer rate for upward flow orientation. Welling and Woolbridge investigated an experimental study to obtain the fin height regarding to the maximum heat transfer rate in the rectangular vertical fins of constant length [24]. Jones and Smith developed a correlation to find the average heat transfer coefficient, and they reported the heat transfer rate from a vertical fin arrays with constant length and thickness [25]. Arquis and Rady studied numerically the effects of the fin spacing, fin height and the Rayleigh number on the fin surface effectiveness [26]. Nada reported the effect of the fin length and fin spacing on the natural convection heat transfer rate from a finned base plate of rectangular shape in horizontal and vertical narrow

enclosures over the wide range of Rayleigh numbers [27]. The effect of the fin height and fin spacing on the heat transfer from rectangular fin attached to horizontal and vertical surfaces [28,29]. Baskaya et al. [30] observed to study the heat transfer rate from horizontal rectangular fin arrays which depend on the fin height, spacing and orientation. Bar-Cohen et al. [31] conducted an optimization technique in which optimum arrays was estimated on the basis of the total dissipation of heat and the heat dissipation per unit mass.

In this study, an optimization of the geometric design parameters in a cylindrical heat sink with branched fins is investigated. The present analysis simulates numerically the heat transfer from circularly arrayed of branched fins on a vertical cylinder under natural convection. The effects of the fin number, fin height, fin length, and the outer diameter of the heat sink on heat transfer are reported at branch angle ($\alpha = 30^\circ$) of fins. To find optimum heat sink design, parametric study is carried out. The reliability of results and the optimum value of each parameter are investigated.

2 MATHEMATICAL MODELING

2.1 Numerical Model

The cylindrical heat sink is composed of a cylindrical base and arrays of the branched fin which are arranged circularly at regular angular intervals. This type of heat sink model is shown in Fig.1. The computational domain of single-fin array is selected owing to the computational time and number of grids involvement. This domain is shown in Fig.2. The following assumptions made for this analysis are given.

- (1) The flow is steady, laminar and three-dimensional.
- (2) Radiation heat transfer is neglected.
- (3) All the fluid properties except the air density are constant.
- (4) The air density is obtained by the ideal gas law.

2.2 Governing Equations

The governing equations are given as follows.

2.2.1 Air Side

Continuity equation

$$\nabla \cdot (\rho v) = 0. \quad (1)$$

Momentum equation

$$\rho \frac{Dv}{Dt} = -\nabla P + \mu \nabla^2 v + F \text{ (for } z \text{ - direction } F = -\rho g).$$

Energy equation

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + \frac{DP}{Dt}. \quad (3)$$

2.2.2 Fin Side

Energy equation

$$\nabla^2 T = 0. \quad (4)$$

2.3 Boundary Conditions

The following boundary conditions are made for this analysis.

(1) Heat sink base of the solid domain: constant heat flux,

$$\dot{q} = -k_s \left. \frac{\partial T_{solid}}{\partial n} \right|_{\text{heat sink base}}.$$

(2) Periodic interface of the solid domain: symmetric condition,

$$\left. \frac{\partial T_{solid}}{\partial n} \right|_{\text{sectional wall}} = 0.$$

(3) Periodic interface of the fluid domain: periodic condition.

(4) Interface between the fluid and solid domain:

$$T_{fluid,wall} = T_{solid,wall},$$

$$-k_f \left. \frac{\partial T_{fluid}}{\partial n} \right|_{wall} + \dot{q}_{out} = -k_s \left. \frac{\partial T_{solid}}{\partial n} \right|_{wall} + \dot{q}_{in}.$$

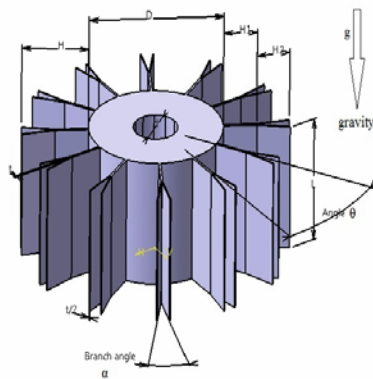


Fig.1. Schematic diagram of the cylindrical heat sink.

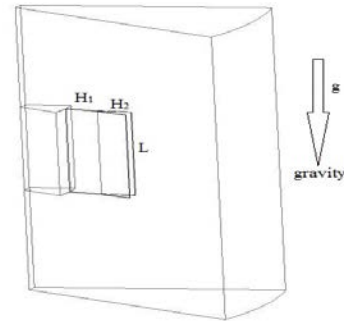


Fig.2. Computational domain.

2.4 Numerical Procedure

For numerical simulation, three-dimensional steady-state laminar flow model was taken to analyze the fluid flow and heat transfer owing to the heat sink under natural convection. The finite volume method with Fluent, a commercial software package of computational fluid dynamics (CFD) was used in this analysis. To governing the conservation of mass, momentum and energy equations, a segregate solver was employed. ICEM CFD 14.5 was applied for boundary conditions to generate the mesh. The finite volume method and the semi implicit method for the pressure linked equation (SIMPLE) algorithm were employed to solve the basic conservation equations. The solver was iterated the procedure to find the solutions for the fluid flow and heat transfer problem until the convergence criteria was satisfied.

2.4.1 Calculation of Numerical Data

In this analysis, the thermophysical properties were calculated at temperature

$$T_m = \frac{T_a + T_b}{2}. \quad (5)$$

where T_a and T_b are the ambient temperature and temperature of the heat sink base respectively.

The total heat transfer from heat sink can written as

$$q = \frac{T_b - T_a}{R_{TH}}. \quad (6)$$

where R_{TH} is the thermal resistance of the heat sink which can be given as

$$R_{TH} = \frac{1}{h(\eta_{fin} N A_f + A_b)}. \quad (7)$$

where η_{fin} , h , A_f , and A_b are the efficiency of fin, heat transfer coefficient, surface area of fin, and surface area of unfinned, respectively.

$$\eta_{fin} = \frac{k_s A_{c1} m_1}{h A_f} \cdot \left[\frac{\cosh m_1 H_1 - C_f}{\sinh m_1 H_1} \right], \quad (8)$$

$$h = Nu_L k_f / L, \quad (9)$$

$$A_f = 2H_1 L + 2H_2 t + 2Ht + Lt, \quad (10)$$

$$A_b = \pi DL - NLt, \quad (11)$$

$$A_{c1} = Lt, \quad (12)$$

$$p_1 = 2L + 2t, \quad (13)$$

$$A_{c2} = Lt/2, \quad (14)$$

$$p_2 = 2L + t, \quad (15)$$

$$m_1 = \sqrt{hp_1/k_s A_{c1}}, \quad (16)$$

$$m_1 = \sqrt{hp_2/k_s A_{c2}}, \quad (17)$$

$$C_f = \left[2 \frac{A_{c2} m_2}{A_{c1} m_1} \sinh m_1 H_1 \cdot \frac{(\sinh m_2 H_2 + h/m_2 k_s \cdot \cosh m_2 H_2)}{(\cosh m_2 H_2 + h/m_2 k_s \cdot \sinh m_2 H_2)} + \cosh m_1 H_1 \right]^{-1}. \quad (18)$$

where Nu_L , A_{c1} , p_1 , A_{c2} , and p_2 are the nusselt number, cross-sectional area of fin, perimeter of fin, cross-sectional area of fin in stem side and perimeter of fin in stem side, respectively.

The number of fins can be calculated as

$$N = \frac{360}{\theta}. \quad (19)$$

Here, the angle θ between the fins was maintained constant at the same lateral spacing regarding the best performance of the heat sink.

3 RESULTS AND ANALYSIS

3.1 Validation of Model

The results of numerical simulation for the reference model at branch angle ($\alpha = 30^\circ$) were taken, as $\theta = 30^\circ$, $D = 60\text{mm}$, $H = 30\text{mm}$, and $L = 50\text{mm}$, and these were compared with experimental data [32]. The temperature difference between the experimental and numerical results was found 1.48°C which exhibited reasonable good agreement, and it can be shown in Fig.3.

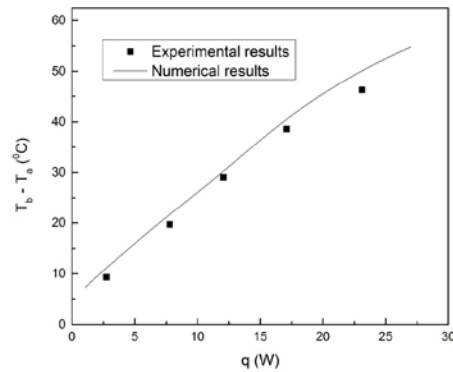


Fig. 3. Comparison between experimental and numerical results at $\alpha = 30^\circ\text{C}$.

3.2 Optimization of Geometric Parameters

Generally, the impact of the heat transfer characteristics is more for the performance of a heat sink. The heat transfer rate of a heat sink depends on the geometric parameters, such as the number of fins, fin height, fin length, fin thickness, and the outer diameter of the heat sink which characterize the heat sink surface. In the present model of a heat sink, the dependent parameter of the heat transfer rate at branch angle ($\alpha = 30^\circ\text{C}$) of fin is the number of fins, fin height, length of fin, and the diameter of the heat sink. This study reported to analyze the effects of the geometric parameters on the performance of the heat sink simultaneously on the basis of the Taguchi approach [33]. In this simulation, the ambient temperature, $T_a = 300\text{K}$ was taken.

The Taguchi method is an effective tool to design system with various parameters which enhances the performance of the reference model, and it is extensively applied in engineering analysis. In this method, a standard table is used called orthogonal arrays (OA) for the design of experiments. Also, its importance is more because of this method reduces the experimental observation time. This method was applied in the present analysis to find optimum condition of the design parameters regarding the best performance of the heat transfer. In this analysis, three geometric parameters as θ , H^* , and L^* , were chosen as control factors, and the number of levels for each control

factor was taken as three. Here, $H^* = H/D$ are $L^* = L/D$ were considered. Table 1 showed the value of control factors and their levels at branch angle ($\alpha = 30^0$) employed in this analysis.

TABLE 1

VALUE OF CONTROL FACTORS AND THEIR LEVELS AT $\alpha = 30^0$ EMPLOYED

Factors	Reference model	Levels of test sample		
		Level 1	Level 2	Level 3
θ	30	9	18	45
H^*	0.5	0.2	0.4	0.6
L^*	0.833	0.7	0.8	0.9

The orthogonal table in the orthogonal array method contained 9 test cases based on three control factors and three levels for each factor. Table 2 indicated a three-level orthogonal array (L_9) used in this analysis. The average heat transfer coefficient for each test sample was estimated from the simulation, and the total heat transfer rate, q was found from Eq. (6).

In the analysis of the Taguchi method, the signal-to-noise (S/N) ratio was employed. In order to estimate the effect of selected factor on the responses, the signal-to-noise ratio for each control factor was calculated. The signals have showed the effect on the average response and the noises were founded by the effect on the deviations from the average responses, and the noises showed the sensitiveness of the experiment output to the noise factors. The appropriate signal-to-noise ratio must be selected. The selection of the control factors and levels were done more efficient manner regarding the heat transfer data was converted into the signal-to-noise ratio, η . In this analysis, the signal-to-noise ratio η for larger-the-better target for all the responses was estimated by the following equation that represents the static characteristics [34].

$$\eta = -10 \log \left[\frac{1}{n} \sum_{i=1}^n \frac{1}{(q)_i^2} \right] \quad (20)$$

where n is number of the design points. The S/N ratio for nine test cases is shown in Table 2 which was obtained by means of MINITAB 17 statistical software.

TABLE 2

L_9 ORTHOGONAL ARRAYS EMPLOYED AND SIGNAL-TO-NOISE (S/N) RATIO FOR EACH TEST AT $\alpha = 30^0$

Test number	Factors			Signal-to-Noise (S/N) ratio
	θ	H^*	L^*	η
1	1	1	1	26.358
2	1	2	2	32.933
3	1	3	3	36.898
4	2	1	2	22.457
5	2	2	3	28.460
6	2	3	1	29.101
7	3	1	3	17.917
8	3	2	1	19.772
9	3	3	2	23.347

TABLE 3

RESPONSE TABLE OF SIGNAL-TO-NOISE RATIOS AND CONTRIBUTION RATIO VALUES

level	Control factors		
	θ	H^*	L^*
η			
1	32.06	22.24	25.08
2	26.67	27.05	26.25
3	20.35	29.78	27.76
Delta, δ			
δ	11.72	7.54	2.68
Contribution ratio (%)			
Contribution (%)	53.42	34.37	12.21

Table 3 is showed the calculated factorial effects and contributions of each factor, and the S/N ratios for each control factor of different levels were found by the average of the S/N ratio values corresponding to each level from Table 2.

The contribution ratio carries the important role to describe the performance characteristics of a heat sink, and it reported to characterize the effect of each factor on the heat transfer rate. The delta δ is the difference between maximum and minimum S/N ratio for each factor, and its role is important to estimate the contribution ratio in this analysis. The contribution ratio of each factor was found from the ratio of the value of delta δ corresponding to each factor to the total value of delta δ [35]. The effect of the factors on the heat transfer rate at $\alpha = 30^\circ$ were found as 53.42%, 34.37%, and 12.21% for θ , H^* , and L^* , respectively in this analysis. It can be clear from Table 3 all three factors affected the performance of heat transfer of the cylindrical heat sink, and also the largest contribution ratio of factor is θ , which showed the maximum effect of this factor on heat transfer performance. It should be noted that the effect of all three factors were limited in this analysis.

For the optimum condition, the S/N ratio from Table 3 for all three factors at $\alpha = 30^\circ$ can be shown in fig. 4. The largest S/N ratio η among three levels for each factor regards to the best thermal performance, and this optimum condition of the heat sink can be estimated from Eq. (20). If angle is considered in this context at $\alpha = 30^\circ$, the optimum results estimated as 9° which is the best results among them. When comparing the S/N ratio η values in Fig. 4, the optimum condition in this analysis was estimated by a combination of levels corresponding to the largest S/N ratio η for each control factor as θ_1 , H_3^* , and L_3^* which is based on the Taguchi method.

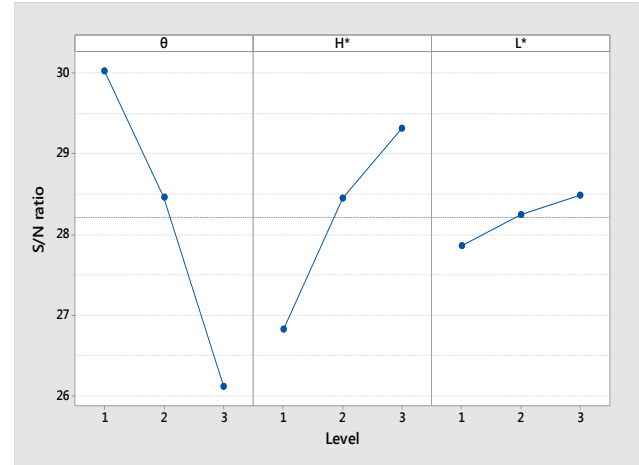


Fig. 4. Response graphs of main effects for all the control factors at $\alpha = 30^\circ$.

3.3 Confirmation Test of the Optimum Model

The combination of the optimum level of each factor was promoted to design the best test samples at $\alpha = 30^\circ$ as $\theta = 9^\circ$, $H/D = 0.6$, and $L/D = 0.9$, respectively. The reliability of the results was tested in this analysis. The presumed ratio η_p of the optimum model was compared with the ratio of optimum sample, and this was estimated by following relation as

$$\eta_p = \eta_{\theta_1} + \eta_{L_3^*} + \eta_{H_3^*} - (m - 1)\bar{\eta} \quad (21)$$

$$\eta_p = 36.88 \text{ at } \alpha = 30^\circ,$$

where m is the total number of factors, $\bar{\eta}$ is the average value of S/N ratio η for all the three factors, and also η_{θ_1} , $\eta_{L_3^*}$, and $\eta_{H_3^*}$ were found from Table 3. The simulation of optimum sample was also estimated by Eq. (20) in which the ratio η at $\alpha = 30^\circ$ was found as 36.90. This value was in good agreement with the presumed value from Eq. (21) that ensured the reliability of the optimum condition.

The total heat transfer rate of the optimum model was compared with the reference model, and the variations of this performance are shown in Fig.5. This variation

reported that the optimum heat sink model showed a better performance than those of the reference heat sink model.

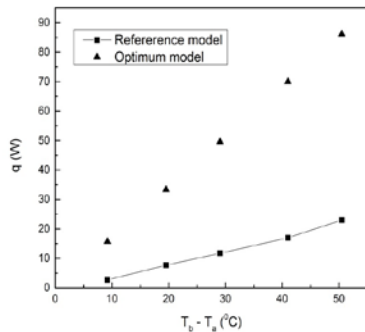


Fig. 5. Variations of heat transfer rate for the optimum and reference models $\alpha=30^0$

4 CONCLUSION

The present model of the heat sink at branch angle ($\alpha=30^0$) reported the effects of various design parameters, as the number of fin, fin height, fin length, and the outer diameter of the heat sink on the heat transfer rate on the basis of the Taguchi approach. In the analysis, it can be shown that the design parameters play an important role in the heat transfer performance of the cylindrical heat sink. The contribution of each tested factor of the heat sink at $\alpha=30^0$ on the heat transfer performance was 53.42%, 34.37%, and 12.21% for θ (or number of fins), H/D, and L/D, respectively in this analysis. The optimum conditions for each factor were found, and the reliability of the optimum results was tested in which good agreement was occurred with the presumed value. The design parameters of the optimum heat sink model was $\theta = 9^0$, H/D = 0.6, and L/D = 0.9. It was also showed that the heat transfer performance of the optimum heat sink model at $\alpha= 30^0$ was superior to that of the reference model.

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